$A=1+10+100+$etc., to $2m$ terms, $=\frac{1}{9}(10^{2m}-1)$.

$B=4+40+400+$etc., to m terms, $=\frac{4}{9}(10^m-1)$.

$A+B+1=\frac{1}{9}(10^{2m}-1)+\frac{4}{9}(10^m-1)+1=\frac{1}{9}(10^{2m}+4.10^m+4)$

$=\{\frac{1}{3}(10+2)\}^2$.

II. **Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas, and H. C. WILKES, Skull Run, West Virginia.**

$A=\frac{9}{16}B^2+\frac{1}{2}B$ as is shown by the following: Let $B=444$.

$\therefore \frac{9}{16}B^2+\frac{1}{2}B=111111$. This is true for any value of B.

Hence $A+B+1=\frac{9}{16}B^2+\frac{1}{2}B+1=\left(\frac{3B+4}{4}\right)^2=B^2$.

$\therefore A+B+1=(333......334)^2$, the number within the parenthesis consists of m figures. Let A_1 be an integer consisting of m figures all 1's.

Then $B^2=(333......334)^2=(B+1+A_1)^2$.

Also solved by *M. A. GRUBER* and *J. SCHEFFER*.

31. **Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.**

How many scalene triangles, of integral sides, can be formed with an altitude of 12? How many isosceles triangles?

I. **Solution by ARTEMUS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington D. C.**

1. To find right-angled triangles having one leg$=12$.

Let $x=$the required leg and $x+a=$the hypothenuse; then $(x+a)^2-x^2=2ax+a^2=12^2=144$; whence $x=\dfrac{144-a^2}{2a}$.

It is easily seen that a must be even, and that it cannot exceed 10; but as x must be integral a can only be 2, 4, 6, or 8.

Take $a=2$, then $x=35$; take $a=4$, then $x=16$; take $a=6$, then $x=9$; take $a=8$, then $x=5$. Hence there are four right-angled triangles having one leg $=12$, viz: 12, 35, 37; 12, 16, 20; 12, 9, 15; 12, 5, 13.

2. Any two right-angled triangles, p, c, a; p, b, d, can be combined in two different ways to form a scalene triangle, giving the triangles a, b, $c+d$; a, b, $c-d$. Hence the four right-angled triangles found above can be combined two and two in two different ways to form scalene triangles; therefore there are twelve such triangles which have an altitude of 12, as follows: 13, 14, 15; 20, 37, 51; 15, 20, 25; 15, 37, 44; 13, 37, 40; 13, 20, 21; 13, 15, 4; 20, 37, 19; 15, 20, 7; 15, 37, 26; 13, 37, 30; 13, 20, 11.

There can be only four isosceles triangles with integral sides having an altitude of 12, viz: 13, 13, 10; 15, 15, 18; 20, 20, 32; 37, 37, 70.

II. **Solution by A. H. BELL, Box 184, Hillsboro, Illinois.**

We evidently require to find two □ numbers whose difference shall be equal to any given number. Let $x=$the side of the lesser square, and $d=$to

two unequal factors$=ab$, $a>b$; let $x+b=$the greater square.

Then $(x+b)^2-x^2=ab$, and $x=\dfrac{a-b}{2}$, $x+b=\dfrac{a+b}{2}$.

The unequal factors of the difference $(12)^2$ are 2×72, 4×36, 6×24, 8×18; these give for sides of squares in the formula, and complete the following right-angled triangles, in the order of altitude, base and hypotenuse: 12, 5, 13; 12, 9, 15; 12, 16, 20; 12, 35, 37.

By doubling the base of each will give four isosceles, and by adding and subtracting the bases from each pair will give 12 scalene triangles.

III. Solution by the PROPOSER.

All scalene $\triangle$'s are rt. $\triangle$'s or are the sum or the difference of two rt. $\triangle$'s of equal altitudes. The $\triangle$'s of this problem are restricted to $\triangle$'s of integral sides having an altitude of 12.

We first find the rt. $\triangle$'s of integral sides having an altitude of 12. These are four in number: 12, 5, 13; 12, 35, 37; 12, 9, 15; and 12, 16, 20.

Then, by *sum* and *difference*, we form combinations by twos by joining their equal altitudes. It will readily be seen, if $n=$the number of rt. $\triangle$'s of a given altitude, that the number of combinations each by *sum* and by *difference* of twos is the sum of the series, $n-1$, $n-2$, $n-3$......1. The sum of this series is $\dfrac{n(n-1)}{2}$. As there are two such series, the number of combinations is $n(n-1)$.

Adding to this the n rt. $\triangle$'s, we find the total number of scalene $\triangle$'s to be n^2, which is the square of the number of rt. $\triangle$'s having the given altitude. Hence the number of scalene $\triangle$'s of integral sides having an altitude of 12 is $4^2=16$.

All isosceles $\triangle$'s of integral sides are the union of two equal rt. $\triangle$'s by joining the altitudes. There are as many isosceles $\triangle$'s of integral sides having a given altitude as there are rt. $\triangle$'s of integral sides having the given altitude. Hence there are four isosceles $\triangle$'s of integral sides having an altitude of 12.

Also solved by *O. W. ANTHONY, H. W. DRAUGHON, G. B. M. ZERR*, and *WILLIAM HOOVER*.

32. Proposed by A. H. BELL, Box 184, Hillsboro, Illinois.

Decompose into its prime factors the number 549755813889.

Solution by the PROPOSER.

To find the factors of $2^{39}+1=549755813889$. The old masters have demonstrated that prime factors of a^n+1 must be of the general form of $2nx+1$. Suppose we take $a^{mn}+1$, mn odd, the factors of mn are m, n, 1; then the prime divisors will be of form $a^{mn}+1$, a^n+1, and $a+1$. Divide out these factors; the $\sqrt{\text{balance}}$ will show the limit of the trial divisors which must be of the general form $2mnx+1=$to prime form of factors$=8mnx+1$ and $8mnx+(6mn+1)$, if these will not or if they do divide the balance, we conclude the balance to be a prime number.

Solution of $2^{39}+1=549{,}755{,}813{,}889\div$by divisors (prime) $2^{13}+1$, 2^3+1,